And Functions

Relation and Function

☆ Cartesian Product

Caretesian product is combination of element of set

If A and B are any two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the Cartesian product of the set A with set B and is denoted by $A \times B$

Thus $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

Remark :-

Two ordered pairs are equal if corresponding element are equal

$(a, b) = (c, d)$

If $a = c$ then $b = d$

2) $n(A) = p$, $n(B) = q$

$n(A \times B) = p \cdot q$

★ Relation

Relation is a subset of Cartesian product

ex. $A \times B = \{(1,a)\ (1,b)\ (2,a)\ (2,b)\}$

$R_1 = \{(1,4)\ (2,4)\}$

$R_2 = \{(1,b)\}$

let $n(A) = M$ $\quad n(B) = N$

$n(A \times B) = M.N$

total Subsets $= 2^{M.N} =$

Note

$R : B \to A$ Relation from B to A

$R : A \to B$ Relation from A to B.

★ Domain (Co-domain and Range)

$R : A \to B$

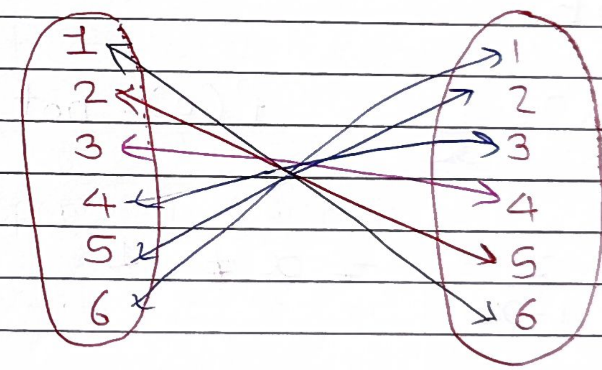
A : Domain

B = Co Domain

Domain is set of all first element relation.

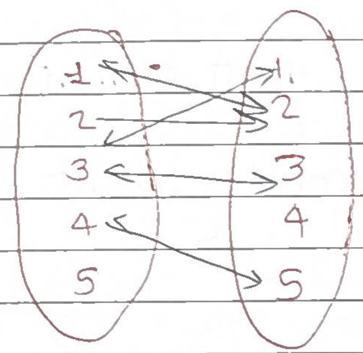
Range is set of all second element relation.

★ Mapping diagram ★

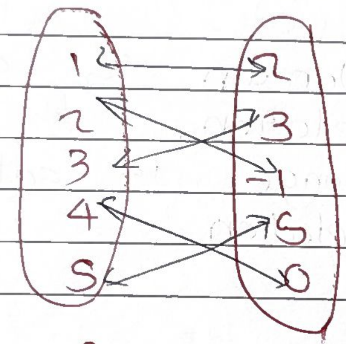


★ function

Def^n : Function is a relation in which every first element have only one image.



not a function



function

$f : A \rightarrow B$

Note : $f(x) = \frac{1}{x}$, $f(0)$ = not defined

$f(x) = \frac{2}{1-x}$ $= x \neq 1$

$\because$ $\sqrt{\text{negative}}$ not difined

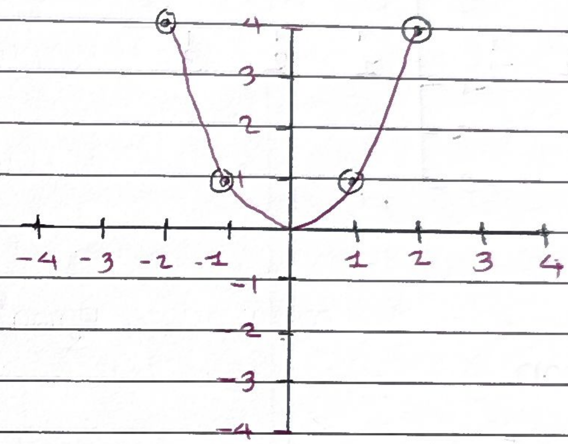
Function is work as machine

Set of all values we can put in function is Domain.

★ Graph of function

$y = f(x) = x^2$

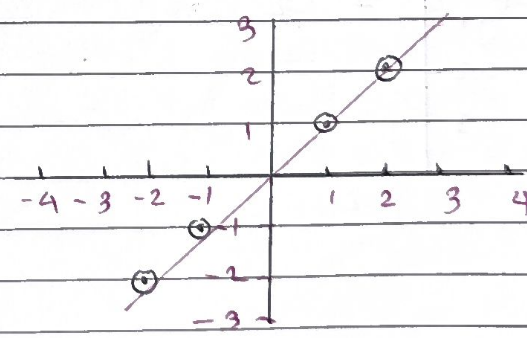
y	4	1	0	1	4
x	2	-1	0	1	2



2) y Indentity function

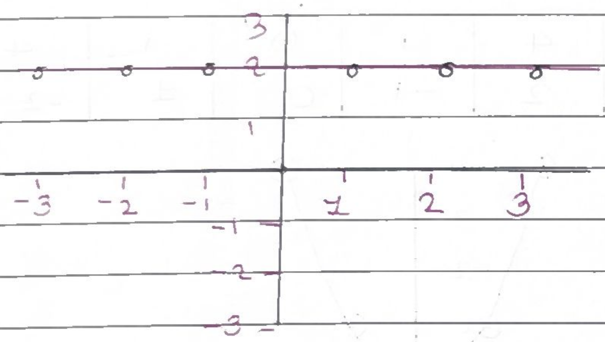
$y = f(x) = x$

x	-2	-1	0	1	2
y	-2	-1	0	1	2



Constant function

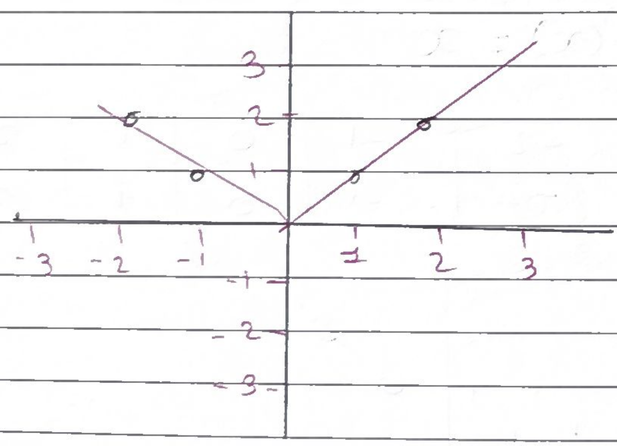
$f(x) = x = 2$



Modulus function

$f(x) = |x|$ $\quad y = |x|$

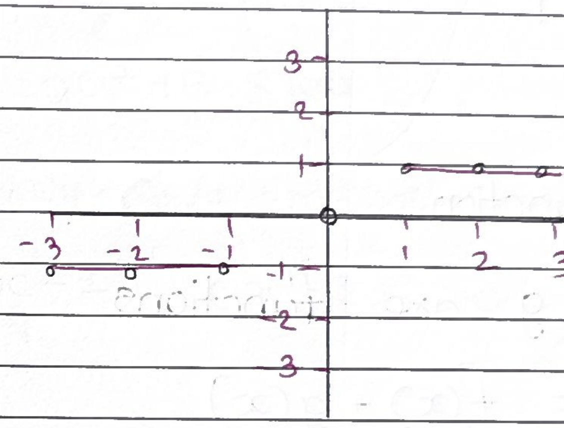
x	=	-2	-1	0	1	2
y	=	2	1	0	1	2



Signum function

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

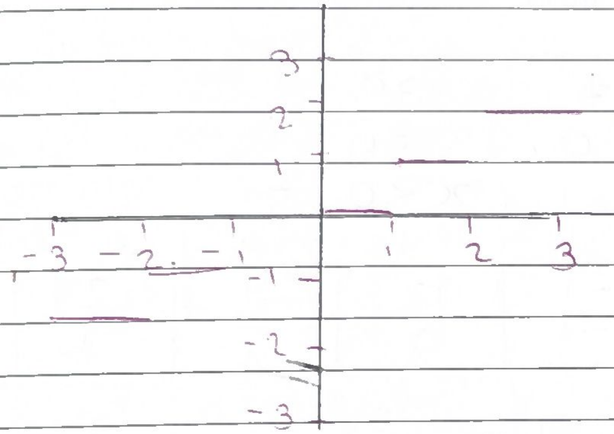
x	-2	-1	0	1	2
y	-1	-1	0	1	1



Greatest integer function

$f(x) = [x]$ $\quad [t] = \{x, x \leq t \leq x\}$

$$[x] = \begin{cases} -2, & -2 < x < -1 \\ -1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \end{cases}$$



Algebra of function

☆ Let f and g are functions

$$(f+g)(x) = f(x) + g(x)$$

☆ Substraction of function

$$(f-g)(x) = f(x) - f(g)$$

☆ Multiplication of functions

$$(f.g)(x) = f(x).f(g)$$

☆ Division of function

$$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x}$$

☆ Scaller multiplication of function

$$(\alpha f)(x) = \alpha . f(x$$

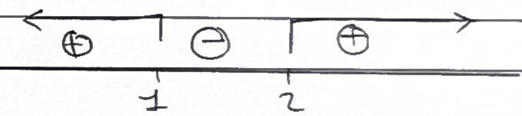
☆ Linear inequality

$$x^2+2 > 5$$

$$x^3+5 \leq 100$$

☆ Wavy Carve method

$$(x-1) \quad (x-2) > 0$$

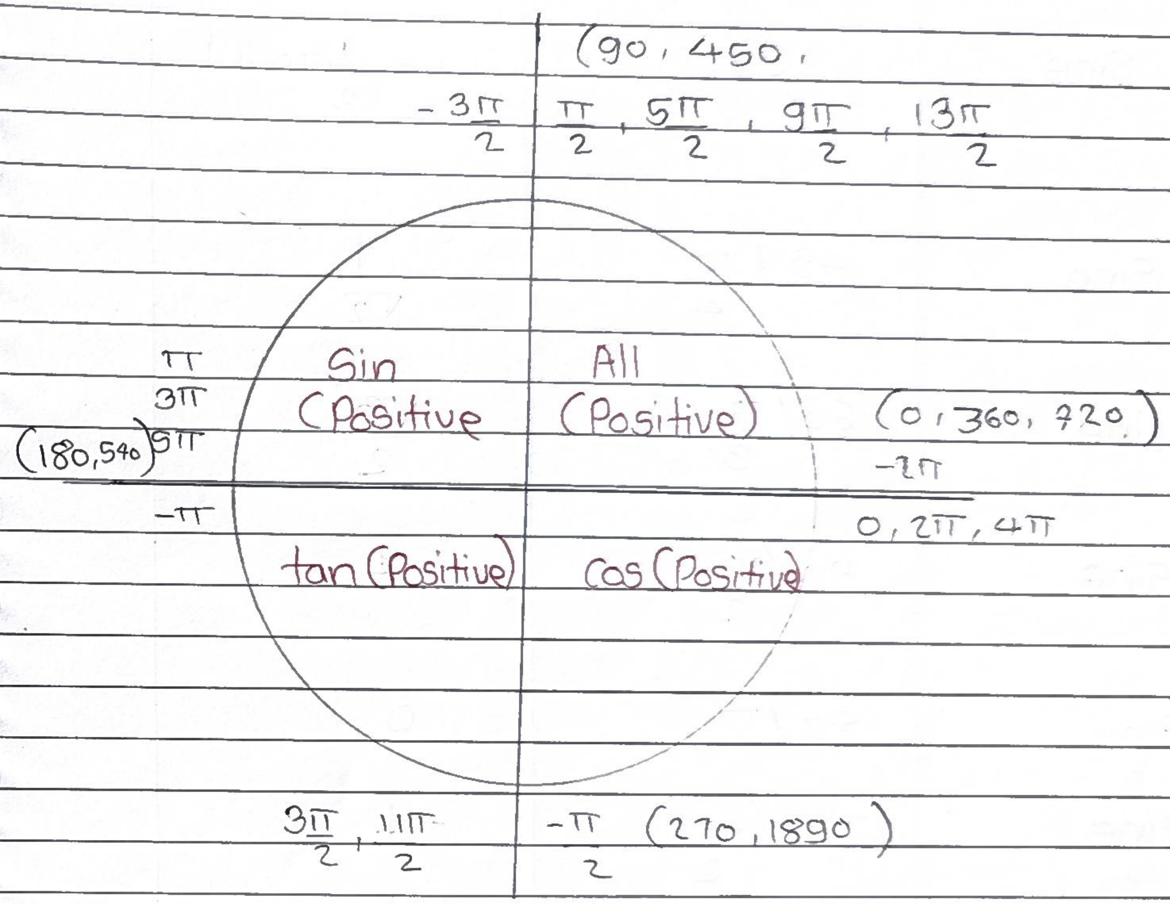


$x = 3$ $= 2$

$x = 4$ $= 6$

☆ Range : Collection of all values of function

Trigonometry



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Angle	Degree and Radian	Value
Sinθ	$30 / \frac{\pi}{6}$	$\frac{1}{2}$
Sinθ	$45 / \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
Sinθ	$60 / \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
Sinθ	$90 / \frac{\pi}{2}$	1
Sinθ	$180 / \pi$	0
Sinθ	$270 / \frac{3\pi}{2}$	-1
Sinθ	$360 / 2\pi$	0
Sinθ	0	0

$\sin 53 = \frac{4}{5}$

$\sin 37 = \frac{3}{5}$

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Angle	Degree and Radian	Value
Cosθ	0	1
Cosθ	$30 / \frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
Cosθ	$45 / \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
Cosθ	$60 / \frac{\pi}{3}$	$\frac{1}{2}$
Cosθ	$90 / \frac{\pi}{2}$	0
Cosθ	$180 / \pi$	-1
Cosθ	$270 / \frac{3\pi}{2}$	0
Cosθ	$360 / 2\pi$	1

$\cos 37 = \frac{4}{5}$

$\cos 53 = \frac{3}{5}$

Angle	Degree and Radian	Value
$\tan\theta$	0	0
$\tan\theta$	$30 / \frac{\pi}{6}$	$\frac{1}{\sqrt{3}}$
$\tan\theta$	$45 / \frac{\pi}{4}$	1
$\tan\theta$	$60 / \frac{\pi}{3}$	$\sqrt{3}$
$\tan\theta$	$90 / \frac{\pi}{2}$	∞
$\tan\theta$	$180 / \pi$	0
$\tan\theta$	$270 / \frac{3\pi}{2}$	∞
$\tan\theta$	$360 / 2\pi$	0

$\tan 37 = \frac{3}{4}$

$\tan 53 = \frac{4}{3}$

* Some Basic formulae

i) $\sin^2 x + \cos^2 x = 1$

ii) $1 + \tan^2 x = \sec^2 x$ or, $\sec^2 x - \tan^2 x = 1$

iii) $\sec x + \tan x = \frac{1}{\sec x - \tan x}$, where $x \neq n\pi + \frac{\pi}{2}$

iv) $1 + \cot^2 x = \operatorname{cosec}^2 x$

$\operatorname{cosec}^2 x - \cot^2 x = 1$

$\operatorname{cosec} x + \cot x = \frac{1}{\operatorname{cosec} x - \cot x}$ where $x \neq n\pi$

Note

$\sin(n\pi + \theta) = \pm \sin\theta$

$\cos(n\pi + \theta) = \pm \cos\theta$

$\tan(n\pi + \theta) = \pm \tan\theta$

$\sin\left((2n+1)\frac{\pi}{2} + \theta\right) = \pm \cos\theta$

$\cos\left((2n+1)\frac{\pi}{2} + \theta\right) = \pm \sin\theta$

$\tan\left((2n+1)\frac{\pi}{2} + \theta\right) = \pm \cot\theta$

$(2n+1)$ is odd

Sum and Differences of two Angle

1. $\sin(x+y) = \sin x.\cos y + \cos x.\sin y$
2. $\sin(x-y) = \sin x.\cos y - \cos x.\sin y$
3. $\cos(x+y) = \cos x.\cos y - \sin x.\sin y$
4. $\cos(x-y) = \cos x.\cos y + \sin x.\sin y$
5. $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x.\tan y}$
6. $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x.\tan y}$
7. $\cot(x+y) = \dfrac{\cot x.\cot y - 1}{\cot x + \cot y}$
8. $\cot(x-y) = \dfrac{\cot x.\cot y + 1}{\cot y - \cot x}$
9. $\sin 2x = 2\sin x.\cos x$
10. $\cos 2x = \cos^2 x - \sin^2 x$
11. $\cos 2x = 2\cos^2 x - 1$
12. $\cos 2x = 1 - 2\sin^2 x$

3. $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

4. $1+\cos 2x = 2\cos^2 x$, $1-\cos 2x = 2\sin^2 x$

5. $\dfrac{1+\cos 2x}{2} = \cos^2 x$, $\dfrac{1-\cos 2x}{2} = \sin^2 x$

6. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$, $x \neq (2n+1)\dfrac{\pi}{4}$

7. $\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}$ $x \neq 2n\pi$

8. $\dfrac{1+\cos x}{\sin x} = \cot\dfrac{x}{2}$ $x \neq (2n+1)\pi$

9. $\dfrac{1-\cos x}{1+\cos x} = \tan^2\dfrac{x}{2}$ $x \neq (2n+1)\pi$

20. $\dfrac{1+\cos x}{1-\cos x} = \cot^2\dfrac{x}{2}$ $x \neq (2n\pi)$

21. $\sin 3x = 3\sin x - 4\sin^3 x$

22. $\cos 3x = 4\cos^3 x - 3\cos x$

23. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

Sum and difference into product

Transformation formulus

1. $\sin A + \sin B = 2\sin\left[\dfrac{A+B}{2}\right].\cos\left[\dfrac{A-B}{2}\right]$

2. $\sin A - \sin B = 2\cos\left[\dfrac{A+B}{2}\right].\sin\left[\dfrac{A-B}{2}\right]$

3. $\cos A + \cos B = 2\cos\left[\dfrac{A+B}{2}\right].\cos\left[\dfrac{A-B}{2}\right]$

4. $\cos A - \cos B = -2\sin\left[\dfrac{A+B}{2}\right].\cancel{\cos}\sin\left[\dfrac{A-B}{2}\right]$

5. $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cdot \cos B}$

6. $\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A \cdot \cos B}$

7. $\cot A + \cot B = \dfrac{\sin(A+B)}{\sin A \cdot \sin B}$

8. $\cot A - \cot B = \dfrac{\sin(A-B)}{\sin A \cdot \sin B}$

9. $2\sin x \cdot \cos y = \sin(x+y) + \sin(x-y)$

10. $2\cos x \cdot \sin y = \sin(x+y) - \sin(x-y)$

11. $2\cos x \cdot \cos y = \cos(x+y) + \cos(x-y)$

12. $2\sin x \cdot \sin y = \cos(x-y) - \cos(x+y)$